nag_prob_chi_sq (g01ecc)

1. Purpose

nag_prob_chi_sq (g01ecc) returns the lower or upper tail probability for the χ^2 distribution with real degrees of freedom.

2. Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_prob_chi_sq(Nag_TailProbability tail, double x, double df,
       NagError *fail)
```

3. Description

The lower tail probability for the χ^2 distribution with ν degrees of freedom, $P(X \leq x : \nu)$, is defined by

$$P(X \leq x : \nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_{0.0}^{x} X^{\nu/2-1} e^{-X/2}\, dX \qquad x \geq 0,\ \nu > 0.$$

To calculate $P(X \leq x : \nu)$ a transformation of a gamma distribution is employed, i.e., a χ^2 distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

4. Parameters

tail
Input: indicates whether the upper or lower tail probability is required.
If **tail** = **Nag_LowerTail**, the lower tail probability is returned, i.e., $P(X \leq x : \nu)$.
If **tail** = **Nag_UpperTail**, the upper tail probability is returned, i.e., $P(X \geq x : \nu)$.
Constraint: **tail** = **Nag_LowerTail** or **Nag_UpperTail**.

x
Input: the value of the χ^2 variate, x, with ν degrees of freedom.
Constraint: **x** ≥ 0.0.

df
Input: the degrees of freedom, ν, of the χ^2 distribution.
Constraint: **df** > 0.0.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

On any of the error conditions listed below except **NE_ALG_NOT_CONV** nag_prob_chi_sq returns 0.0.

NE_BAD_PARAM
On entry, parameter **tail** had an illegal value.

NE_REAL_ARG_LT
On entry, **x** must not be less than 0.0: **x** = ⟨*value*⟩.

NE_REAL_ARG_LE
On entry, **df** must not be less than or equal to 0.0: **df** = ⟨*value*⟩.

NE_ALG_NOT_CONV
The series used to calculate the gamma probabilities has failed to converge.
The result returned should represent an approximation to the solution.

6. Further Comments

For higher accuracy the transformation described in Section 3 may be used with a direct call to nag_incomplete_gamma (s14bac).

6.1. Accuracy

A relative accuracy of 5 significant figures is obtained in most cases.

6.2. References

Abramowitz M and Stegun I A (1965) *Handbook of Mathematical Functions* Dover Publications, New York.
Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth.

7. See Also

None.

8. Example

Values from various χ^2 distributions are read, the lower-tail probabilities calculated, and all these values printed out, until the end of data is reached.

8.1. Program Text

```
/* nag_prob_chi_sq(g01ecc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
  double df, prob, x;
  static NagError fail;

  /*  Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("g01ecc Example Program Results\n");
  Vprintf("  x      df     prob\n\n");
  while (scanf("%lf %lf", &x, &df) != EOF)
    {
      prob = g01ecc(Nag_LowerTail, x, df, &fail);
      if (fail.code==NE_NOERROR)
        Vprintf("%6.3f%8.3f%8.4f\n", x, df, prob);
      else
        {
          Vprintf("%6.3f%8.3f%8.4f\n Note: %s\n", x, df, prob, fail.message);
          exit(EXIT_FAILURE);
        }
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01ecc Example Program Data
  8.26   20.0
  6.2     7.5
 55.76   45.0
```

8.3. Program Results

```
g01ecc Example Program Results
  x       df      prob

 8.260  20.000  0.0100
 6.200   7.500  0.4279
55.760  45.000  0.8694
```